PROFIT ANALYSIS OF A TWO – UNIT COLD STANDBY SYSTEM WITH INSTRUCTION AND PREPRATION TIME FOR REPAIR

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ABSTRACT

The cost benefit analysis of a two unit cold standby system with instruction and preparation time for repair is carried out. The repair man (who is an expert) does not come alone but he comes with his assistant. If the expert repair man is busy in repairing a failed unit and second unit fails then the assistant repairman repairs the latter unit after getting instructions from expert repairman. Various measures of system effectiveness are found using regenerative point technique. Graphs are plotted for a special case.

KEYWORDS

Regenerative points, Markov Renwal Process, Reliability, Instruction time, Preparation time, Expert Repairman and Assistant Repairman.

INTRODUCTION

Many investigations concerning the reliability of two-unit system are being made. Various authors including [6-8] discussed two-unit cold standby systems assuming that whenever the operative unit fails, it goes under repair immediately i.e. preparation time for repair is negligible. Guo Tong De [9] and some others studied two-unit systems with preparation time for repair and with one repairman serving at a time. However, practically there may be situations when on failure of a unit the expert repairman does not come alone but he comes with his assistant. The assistant repairman repairs the failed unit only after getting instructions from expert which are given at the time of system failure so that both the failed unit go under repair of both the repairman – one under expert and the other under his assistant.

So, in this paper, we have investigated a two-unit cold stand by system with instruction and preparation time for repair whenever a unit fails, we call an expert repairman immediately to repair the failed unit. He comes with his assistant who does the repair of failed unit perfectly only if instructions are given to him by the expert. The expert repairman first repair himself for repair i.e. he makes the arrangement of tools and other essential materials which are necessary for repair the failed unit and then he starts repair. If a unit is under repair/preparation for repair of the expert and at that time a second unit fails the repairman leaves the repair/preparation for repair of former unit and starts giving instruction to his assistant. It is assumed that after getting instructions the assistant repairman repairs perfectly. And one unit goes under repair of expert and other under repair of his assistant. If at the time when both the repairman are busy, the expert completes the repair earlier than his assistant then he takes the unit, which was under the repair of his assistant, under his repair. The system is analysed by using regenerative point in Markov-Renewal Process and obtained the various of system effectiveness such as mean time to system failure (MTSF), steady-state availability of the system, expected busy period of the repair facility, expected number of visits by repair facility in steady-state. Using the above measure expected profit incurred to the system is also calculated in steady-state. Graphs are plotted for a special case.

MODEL DESCRIPITION AND ASSUMPTIONS

(i) The system consists of two identical units, initially one is operative and the other is kept as cold standby.

(ii) Each unit of the system has two modes – normal and complete failures.

(iii) The failure of a unit, the repair time of assistant repairman are assumed exponentially distributed whereas the instructions time, preparation time for repair and repaired time for expert are arbitrary distributed.

(iv) The payment for the costs of repairs and visits are made to expert only and not to his assistant. The expert pays himself to his assistant.

(v) After any repair a unit becomes like a new one.

(vi) Failures are self announcing and switching is perfect and instantaneous.

(vii) All the random variables are mutually independent.

(viii) If both the repairman are preparing for repair, their preparation time complete simultaneously.

(ix) when expert takes the unit from his assistant for repair, there is no need of preparation time because the type of failure has already been detected and the arrangement & other essential material has already been made.

Other assumptions are already mentioned in the introduction.

NOTATIONS

o/cs operative/cold-standby

F_{up} unit in F mode and under preparation for repair

F_{re}/F_{ra} unit in F-mode and under repair of expert/assistant.

F_{Re} unit in F-mode and repair is continued by an expert repairman from earlier state.

F_{wri} unit in F-mode and waiting for repair when instructions are being given to assistant.

λ Constant failure rate of an operative unit.

μ Constant repair rate of assistant repairman.

h(t), H(t) pdf and cdf of time to preparation for repair of failed unit.

g(t), G(t) pdf and cdf of time to repair by expert.

i(t), I(t) pdf and cdf of time when expert gives instructions to his assistant.

For rest of the notations see reference [10].

Thus considering the above symbols, possible states of the system and the transitions into the states are shown in Fig. 1. The epoch of entrance into the states 6 from 5 is non regenerative.

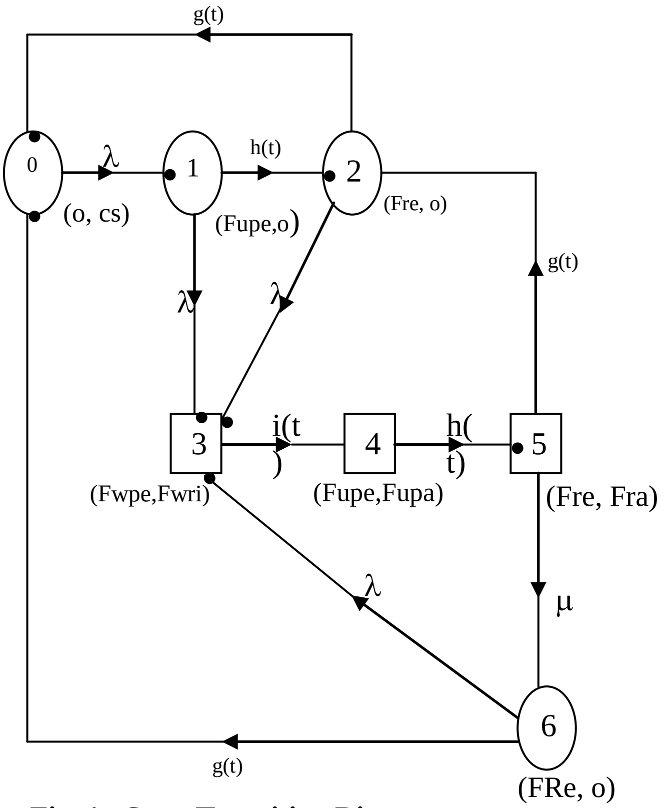

Fig. 1 : State Transition Diagram.

Up State

Failed State

Regenerative Point

TRANSITION PROBABILITIES AND SOUJOURN TIMES

The non-zero elements p_{ij} of the transition probability matrix (t.p.m.) for the system are as follows :

$$p_{01} = 1,\ \ p_{12} = h^*(\lambda),\ \ p_{13} = 1 - h^*(\lambda),\ \ p_{20} = g^*(\lambda)$$

$$p_{23} = 1 - g^*(\lambda),\quad p_{34} = 1,\ \ p_{45} = 1,\ \ p_{52} = g^*(\mu),$$

$$p_{50}^{(6)} = \mu[g^*(\mu) - g^*(\lambda)]/(\lambda-\mu),$$

$$p_{53}^{(6)} = [\mu g^*(\lambda) - \lambda g^*(\mu) + (\lambda-\mu)]/(\lambda-\mu)$$

By these transition probabilities, it can be verified that

$$p_{01} = p_{34} = p_{45} = 1, \qquad p_{20} + p_{23} = p_{12} + p_{13} = 1$$

$$p_{52} + p_{50}^{(6)} + p_{53}^{(6)} = 1. \qquad (1\text{-}3)$$

Let T_i be the sojourn time in state S_i and $\mu_i = E(T_i)$, then using the formula

$\mu_i = \int p(T_i > t)dt$

then for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{Q_{01}^{**}(s)[Q_{13}^{**}(s) + Q_{12}^{**}(s)Q_{23}^{**}(s)]}{1 - Q_{01}^{**}(s)Q_{12}^{**}(s)Q_{20}^{**}(s)} \qquad (4)$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} \qquad (5)$$

Using L′ Hospital's rule and substituting the value of $\phi_0^{**}(s)$ from equation (4), we have

$$T_0 = \frac{\mu_0 + \mu_1 + \mu_2 p_{12}}{1 - p_{12}p_{20}} \qquad (6)$$

AVAILABILITY ANALYSIS

As defined, $M_i(t)$ denotes the probability that the system starting in up state (regenerative state) is up at time t without passing through any regenerative state. Thus we have

$$M_0(t) = e^{-\lambda t}, \qquad M_1(t) = e^{-\lambda t}\bar{H}(t)$$

$$M_2(t) = e^{-\lambda t}\bar{G}(t), \qquad M_5(t) = \left[\mu e^{-\mu t} © e^{-\lambda t}\right]\bar{G}(t)$$

(7-10)

and

$AV_0(t) = M_0(t) + q_{01}(t) © AV_1(t)$

$AV_1(t) = M_1(t) + q_{12}(t) © AV_2 + q_{13}(t) © AV_3(t)$

$AV_2(t) = M_2(t) + q_{20}(t) © AV_0 + q_{23}(t) © AV_3(t)$

$AV_3(t) = q_{34}(t) © AV_4(t)$

$AV_4(t) = q_{45}(t)$ © $AV_5(t)$

$AV_5(t) = M_5 + q_{52}(t)$ © $AV_2 + q_{50}^{(6)}$ © $AV_0(t) + q_{53}^{(6)}$ © $AV_3(t)$ (11-16)

Taking the Laplace transform of the above equations and solving them for $AV_0^*(s)$ and then steady-state availability of the system is given by

$$AV_0 = \lim_{t\to\infty} AV_0(t) = \lim_{s\to 0} AV_0^*(s) = \frac{N_1}{D_1} \quad (17)$$

where

$$N_1 = (\mu_0+\mu_1)(1-p_{53}^{(6)}-p_{23}p_{52}) + \mu_2(p_{12}p_{50}^{(6)}+p_{52}) + \varepsilon_1(1-p_{12}p_{20}) \quad (18)$$

where $M_5^*(0) = \varepsilon_1 = \mu(\mu_5-\mu_2)/(\lambda-\mu)$ (say)

$$D_1 = (\mu_2+(\mu_0+\mu_1)p_{20})p_{52} + (\mu_0+\mu_1+\mu_2 p_{12})p_{50}^{(6)} + (\mu_3+\mu_4+m_1)(1-p_{12}p_{20}) \quad (19)$$

where

$$m_1 = m_{52} + m_{50}^{(6)} + m_{53}^{(6)} = (\lambda\mu_5 - \mu\mu_2)/(\lambda-\mu)$$

BUSY-PERIOD ANALYSIS OF AN EXPERT REPAIRMAN

$B_0(t) = q_{01}(t)$ © $B_1(t)$

$B_1(t) = W_1(t) + q_{12}(t)$ © $B_2(t) + q_{13}(t)$ © $B_3(t)$

$B_2(t) = W_2(t) + q_{20}(t)$ © $B_0(t) + q_{23}(t)$ © $B_3(t)$

$B_3(t) = W_3(t) + q_{34}(t)$ © $B_4(t)$

$B_4(t) = W_4(t) + q_{45}(t)$ © $B_5(t)$

$B_5(t) = W_5(t) + q_{52}(t)$ © $B_2(t) + q_{50}^{(6)}(t)$ © $B_0(t) + q_{53}^{(6)}(t)$ © $B_3(t)$ (20-25)

where

$$W_1(t) = e^{-\lambda t}\bar{H}(t), \qquad W_2(t) = e^{-\lambda t}\bar{G}(t), \qquad W_3(t) = \bar{I}(t)$$

$$W_4(t) = \bar{H}(t), \qquad W_5(t) = e^{-\mu t}\bar{G}(t) + [\mu e^{-\mu t} © e^{-\lambda t}]\bar{G}(t)$$

Solving equations (20-25) for $B^*(s)$ with the help of Laplace-transform and then in a steady-state, the total fraction of the time for which the expert repairman is busy, is given by

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_2}{D_1} \quad (26)$$

$$N_2 = \mu_1(1-p_{53}^{(6)}-p_{23}p_{52}) + \mu_2(p_{52}+p_{12}p_{50}^{(6)})(\mu_3+\mu_4+\varepsilon_1)(1-p_{12}p_{20}) \quad (27)$$

and D_1 is already specified.

EXPECTED NUMBER OF VISITS BY THE EXPERT REPAIRMAN

$V_0(t) = Q_{01}(t) \circledS [1+ V_1(t)]$

$V_1(t) = Q_{12}(t) \circledS V_2(t) + Q_{13}(t) \circledS V_3(t)$

$V_2(t) = Q_{20}(t) \circledS V_0(t) + Q_{23}(t) \circledS V_3(t)$

$V_3(t) = Q_{34}(t) \circledS V_4(t)$

$V_4(t) = Q_{45}(t) \circledS V_5(t)$

$V_5(t) = Q_{52}(t) \circledS V_2(t) + Q_{50}^{(6)}(t) \circledS V_0(t) + Q_{53}^{(6)}(t) \circledS V_3(t)$ (28-33)

Taking Laplace-Stieltjes transform and solving the above equations for $V_0^{**}(s)$ and in steady-state the number of visits per unit time is given by

$$V_0 = \lim_{t\to\infty}\left[\frac{V_0(t)}{t}\right] = \lim_{s\to 0} s.V_0^{**}(s) = \frac{N_3}{D_1} \quad (34)$$

where

$$N_3 = p_{20}p_{52} + p_{50}^{(6)} \quad (35)$$

and D_1 is already specified.

COST-BENEFIT ANALYSIS

The expected total profit incurred to the system in a steady-state is

$$P = K_0AV_0 - K_1B_0 - K_2V_0 \quad (36)$$

where K_0 is the revenue per unit up-time of the system, K_1 is the cost per unit time for which the expert repairman is busy, K_2 is the cost per unit visits by the expert repairman.

STUDY OF SYSTEM BEHAVIOUR THROUGH GRAPHS

For a graphical representation the following particular case is considered.

$h(t) = \alpha \exp(-\alpha t)$
$g(t) = \beta \exp(-\beta t)$
$i(t) = \gamma \exp(-\gamma t)$

For a concrete study of MTSF and profit function, we plot these characteristics w.r.t. λ (the failure rate of an operative unit) for different values of α (the repair rate of an expert). The curve so obtain are shown in Fig. 2 and 3, respectively.

In Fig. 2 each curve represents the graph between λ and MTSF for $\alpha = 3$ and $\alpha = 5$. While the other parameters are constant. From these curves, we observe that the MTSF decreases with an increase in λ. However, for increasing values of α, the MTSF tends to be higher.

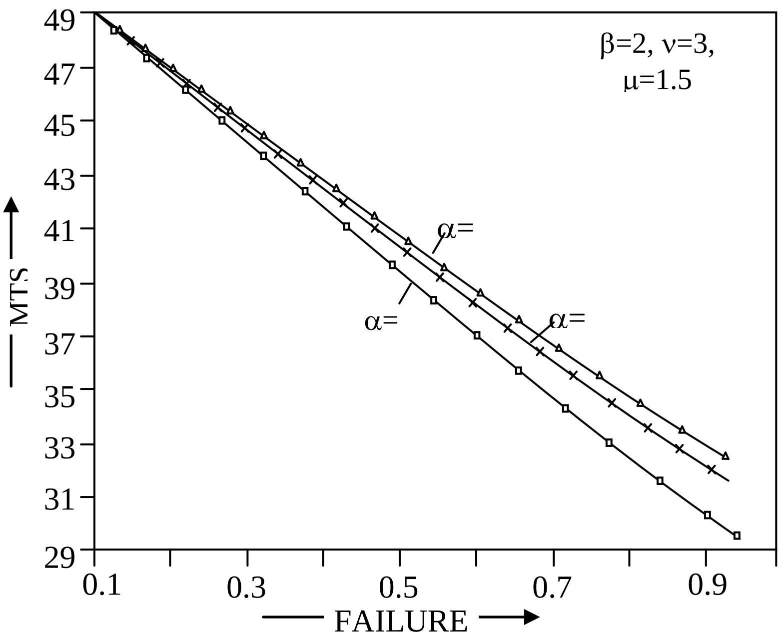

Fig. 2 : Behaviour of MTSF with respect to Failure Rate (λ)

Fig. 3 represents the behaviour of profit function w.r.t. λ for α =3, 4 and 5 while the other parameters are kept constant. It is observed that there is a uniform decrease in profit with an increase in λ and it increases with an increase in α.

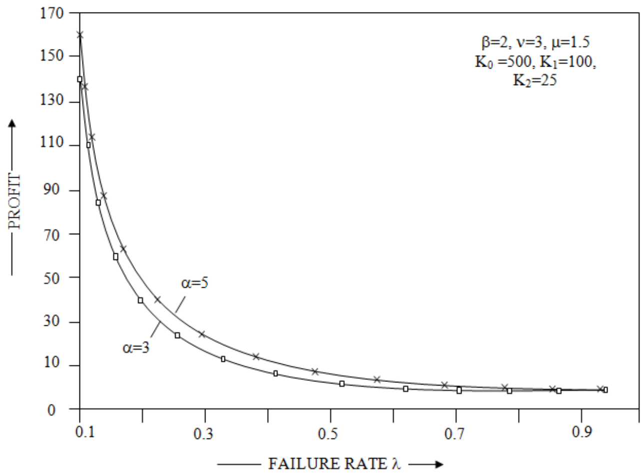

Fig. 3 : Behaviour of Profit with respect to Failure Rate (λ)

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