A SYSTEM WITH GUARANTEE PERIOD AND TWO TYPES OF REPAIR

SURESH KUMAR GUPTA
Dept. of Mathematics
Maharaja Agrasen Institute of Technology,
Sec.-22, Rohini, Delhi (India)

ABSTRACT

A two-unit cold standby system with guarantee period and two types of repair is analysed and profit is evaluated. If a unit fails before completion of guarantee period (after completion of guarantee period) then it is repaired by manufacturer (management) at his own cost. The system is analysed using semi-Markov processes and regenerative processes. Various measures of system effectiveness are determined.

KEYWORDS

Regenerative points, Markov Renwal Process (MRP), Reliability, Guarantee Period, Company Repairman and Own Repairman.

INTRODUCTION

Taking into consideration various aspects such as failures, repairs, inspections, costs, alternate periods, etc., many papers have been appeared to study one or two-unit reliability systems. In most of these papers, such as [6-10] it is assumed that whenever a unit fails, it goes to repairman for repair immediately or after some wait. The management has to pay for the busy-period/visits of the repairman. However, there may be units/systems for which guarantee for certain fixed period is given by the manufacturer. A unit, if failed during guarantee period, is repaired by the manufacturer at his cost while for a unit which fails after the guarantee period we have to employ maintenance crew at our own cost. This type of situation has not been discussed so far in the literature of reliability and our aim is to fill in such a gap.

Keeping the above observations in view, we study, in this paper, a two-unit cold standby system with guarantee period for each unit. Both the units are purchased and put into the system at the same time. Whenever a unit fails during guarantee period, it is repaired by manufacturer/company at its own cost and if it fails after completion of guarantee period one's own repairman is engaged to repair it. If during a time, the repair of a failed unit is continuing by the company and the guarantee period finishes; the company leaves the unit only after completing its repair without taking any charge for this.

By identifying suitable regenerative points, the expressions for various measures of system effectiveness such as mean time to system failure, steady-state availability, total fraction of busy-time of repairman and expected number of visits by him are determined. Using the above measures profit analysis is made. Graphs pertaining to a particular case are plotted.

NOTATIONS AND STATES OF THE SYSTEM

λ Constant failure rate

μ Constant repair rate of company

a Probability that unit fails during guarantee period

b Probability that unit fails after guarantee period

g(t), G(t) pdf and cdf of repair time when the repair is done by our repairman

m $\int t dG(t)$

For the states of the system, we define the following symbols :

o/cs Operative/cold standby

gf guarantee of the system finishes

F_{Ir} unit in F-mode and under repair of company repairman

F_{IIr} unit in F-mode and under repair of our repairman

F_{wr} Unit in F-mode and waiting for repair.

Using the above symbols, the possible states of the system and the transitions between them alongwith failure of repair rates are shown in Fig. 1. For rest of the notations see[6].

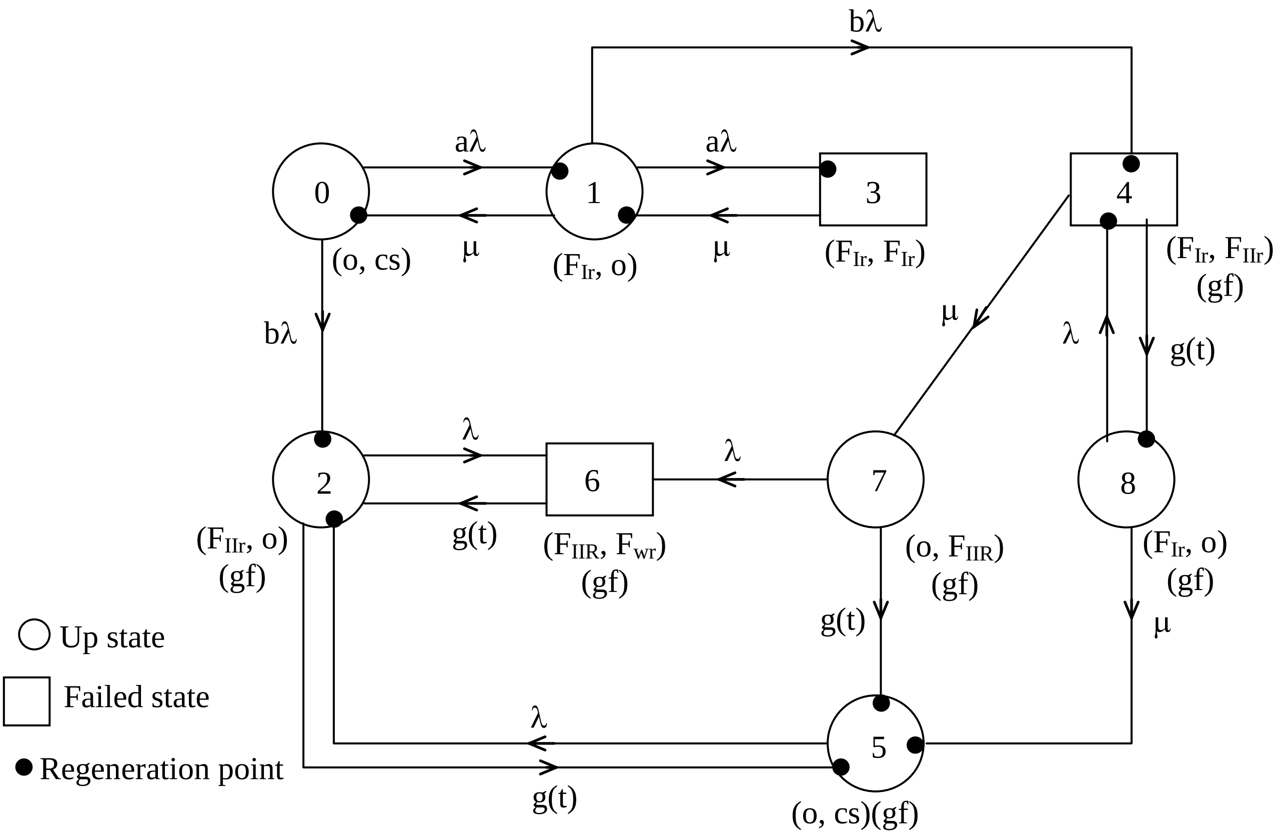

Fig. 1 : State Transition Diagram

TRANSITION PROBABILITIES AND SOJOURN TIMES

The non-zero elements p_{ij} of the transition probability matrix (t.p.m.) for the model are as follows :

$$p_{01} = a, p_{02} = b$$

$$p_{10} = \mu/(\lambda+\mu), p_{13} = a\lambda/(\lambda+\mu), p_{14} = b\lambda/(\lambda+\mu)$$

$$p_{25} = g^*(\lambda), p_{22}^{(6)} = p_{26} = 1 - g^*(\lambda).$$

$$p_{31} = p_{52} = 1, p_{48} = g^*(\mu)$$

$$p_{45}^{(7)} = \mu(g^*(\mu) - g^*(\lambda))/(\lambda-\mu)$$

$$p_{42}^{(7,6)} = (-\lambda g^*(\mu) + \mu g^*(\lambda)) + (\lambda-\mu)/(\lambda-\mu)$$

$$p_{84} = \lambda/(\lambda+\mu), p_{85} = \mu/(\lambda+\mu)$$

It can easily be verified that

$$p_{01} + p_{02} = 1, p_{10} + p_{13} + p_{14} = 1, p_{25} + \left(p_{22}^{(6)} = p_{26}\right) = 1$$

$$p_{31} = p_{52} = 1, p_{48} + p_{45}^{(7)} + p_{42}^{(7,6)} = 1, p_{84} + p_{85} = 1$$

Using the formula

$$\mu_i = \int p(T_i > t)dt,$$

for the mean sojourn time in state S_i, its values for various states are

$$\mu_0 = \mu_5 = 1/\lambda, \mu_1 = \mu_8 = 1/(\lambda+\mu), \mu_2 = [1 - g^*(\lambda)]/\lambda$$

$$\mu_3 = 1/2\mu, \mu_4 = [1 - g^*(\mu)]/\mu$$

MEAN TIME TO SYSTEM FAILURE

To determine the MTSF of the system we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$$\phi_0(t) = Q_{01}(t) \textcircled{s} \phi_1(t) + Q_{02}(t) \textcircled{s} \phi_2(t)$$

$$\phi_1(t) = Q_{10}(t) \textcircled{s} \phi_0(t) + Q_{13}(t) + Q_{14}(t)$$

$$\phi_2(t) = Q_{25}(t) \textcircled{s} \phi_5(t) + Q_{26}(t)$$

(1-3)

Solving the above system of equations (1-3) by using the technique of L.S.T., one can easily get the expression for $\phi_0(t)$ in terms of its L.S.T. i.e. $\phi_0^{**}(s)$. Now the MTSF, given that the system started at the beginning of state S_0, is

$$T_0 = \lim_{s\to 0}(1-\phi_0^{**}(s))/s = \frac{N_1}{D_1}$$

(4)

where

$N_1 = (\mu_0 + \mu_1)\, p_{01}p_{26} + (\mu_0 + \mu_2)\, p_{02}$

and

$D_1 = (1 - p_{01}p_{10})p_{26}$.

AVAILABILITY ANALYSIS

$AV_0(t) = M_0(t) + q_{01}(t) © AV_1(t) + q_{02}(t) © AV_2(t)$

$AV_1(t) = M_1(t) + q_{10}(t) © AV_0(t) + q_{13}(t) © AV_3(t) + q_{14}(t) © AV_4(t)$

$AV_2(t) = M_2(t) + q_{25}(t) © AV_5(t) + q_{22}^{(6)}(t) © AV_2(t)$

$AV_3(t) = q_{31}(t) © AV_1(t)$

$AV_4(t) = M_4(t) + q_{48}(t) © AV_8(t) + q_{42}^{(7,6)}(t) © AV_2(t) + q_{45}^{(7)}(t) © AV_5(t)$

$AV_5(t) = M_5(t) + q_{52}(t) © AV_2(t)$

$AV_8(t) = M_8(t) + q_{84}(t) © AV_4(t) + q_{85}(t) © AV_5(t)$

(5-11)

where

$$M_0(t) = M_5(t) = e^{-\lambda t}$$

$$M_1(t) = M_8(t) = e^{-(\lambda+\mu)t}$$

$$M_2(t) = e^{-\lambda t}\bar{G}(t)$$

$$M_4(t) = \left[\mu e^{-\mu t} © e^{-\lambda t}\right]\bar{G}(t)$$

Taking the L.T. of the above equations and solving them for $A_0^*(s)$ and then steady-state availability of the system is given by

$$AV_0 = \lim_{t\to\infty} s\, AV_0^*(s) = \frac{N_2}{D_2}$$

where

$$N_2 = (1 - p_{48}p_{84})(p_{02}p_{10} + p_{14})(\mu_0 p_{25} + \mu_2)$$

and

$$D_2 = (1 - p_{48}p_{84})(p_{02}p_{10} + p_{14})(\mu_0 p_{25} + m)$$

BUSY PERIOD ANALYSIS OF THE REPAIRMAN

$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t)$

$B_1(t) = q_{10}(t) © B_0(t) + q_{13}(t) © B_3(t) + q_{14}(t) © B_4(t)$

$B_2(t) = W_2(t) + q_{25}(t) © B_5(t) + q_{22}^{(6)}(t) © B_2(t)$

$B_3(t) = q_{31}(t) © B_1(t)$

$B_4(t) = W_4(t) + q_{48}(t) © B_8(t) + q_{42}^{(7,6)}(t) © B_2(t) + q_{45}^{(7)}(t) © B_5(t)$

$B_5(t) = q_{52}(t) © B_2(t)$

$B_8(t) = q_{84}(t) © B_4(t) + q_{85}(t) © B_5(t)$

(12-18)

where

$$W_2(t) = e^{-\lambda t}\bar{G}(t) + \left[\lambda e^{-\lambda t} © 1\right]\bar{G}(t)$$

and

$$W_4(t) = e^{-\mu t}\bar{G}(t) + \left[\mu e^{-\mu t} © e^{-\lambda t}\right]\bar{G}(t) + \left[\mu e^{-\mu t} © \lambda e^{-\lambda t} © 1\right]\bar{G}(t)$$

Solving the equations (12-18) for $B_0^*(s)$ with the help of L.T. and then in a steady-state, the total fraction of the time for which the repairman is busy is given by

$$B_0 = \lim_{s\to 0} s\, B_0^*(s) = \frac{N_3}{D_2}$$

where

$$N_3 = (1 - p_{48}p_{84})(p_{02}p_{10} + p_{14})m$$

and D_2 is already specified.

EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

$$V_0(t) = Q_{01}(t) Ⓢ V_1(t) + Q_{02}(t) Ⓢ [1 + V_2(t)]$$

$$V_1(t) = Q_{10}(t) Ⓢ V_0(t) + Q_{13}(t) Ⓢ V_3(t) + Q_{14}(t) Ⓢ [1 + V_4(t)]$$

$$V_2(t) = Q_{25}(t) Ⓢ V_5(t) + Q_{22}^{(6)}(t) Ⓢ [1 + V_2(t)]$$

$$V_3(t) = Q_{31}(t) Ⓢ V_1(t)$$

$$V_4(t) = Q_{48}(t) Ⓢ V_8(t) + Q_{45}^{(7)}(t) Ⓢ V_5(t) + Q_{42}^{(7,6)}(t) Ⓢ [1 + V_2(t)]$$

$$V_5(t) = Q_{52}(t) Ⓢ [1 + V_2(t)]$$

$$V_8(t) = Q_{85}(t) Ⓢ V_5(t) + Q_{84}(t) Ⓢ [1 + V_4(t)]$$

(20-26)

Taking the L.S.T. of the equations (20-26) and solving then for $V_0^{**}(s)$ and then in a steady-state the number of visits per unit time is given by

$$V_0 = \lim_{t\to\infty}\left[\frac{V_0(t)}{t}\right] = \lim_{s\to 0} s.V_0^{**}(s) = \frac{N_4}{D_2}$$

(27)

where

$$N_4 = (1 - p_{48}p_{84})(p_{02}p_{10} + p_{14})$$

and D_2 is already specified.

PROFIT ANALYSIS

The expected profit of the system in a steady-state is given by

$$P = K_0A_0 - K_1B_0 - K_2V_0$$

where

K_0 = revenue per unit up-time of the system,

K_1 = cost per unit time for which the repairman is busy,

K_2 = cost per unit time for visits by the repairman.

PARTICULAR CASE

Let the repair time distribution is exponential, i.e.,

$$g(t) = \alpha \exp(-\alpha t)$$

To show the trend of the various parameters, we have plotted the graphs which show the behaviour of :

(i) MTSF w.r.t. failure rate λ (keep other parameters as fixed) in Fig. 2.

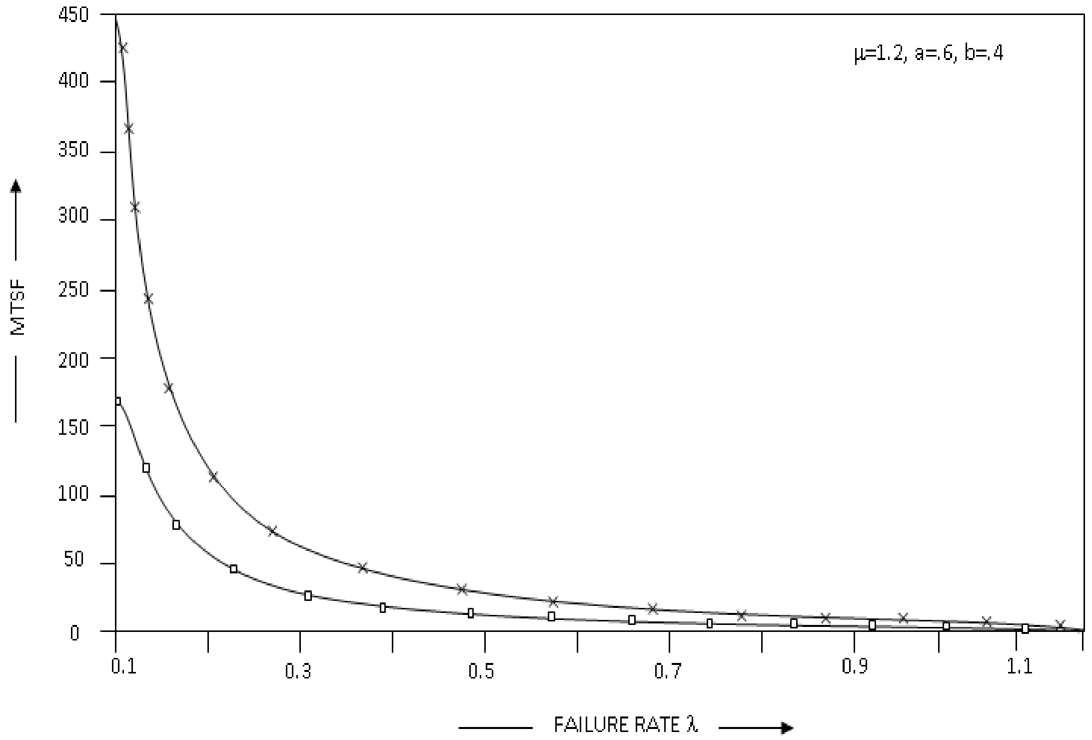

Fig. 2. (□)λ = 1.6, (×) α = 4.6

(ii) MTSF w.r.t. repair rate of company μ for different values of failure rate (keeping other parameters as fixed) in Fig 3.

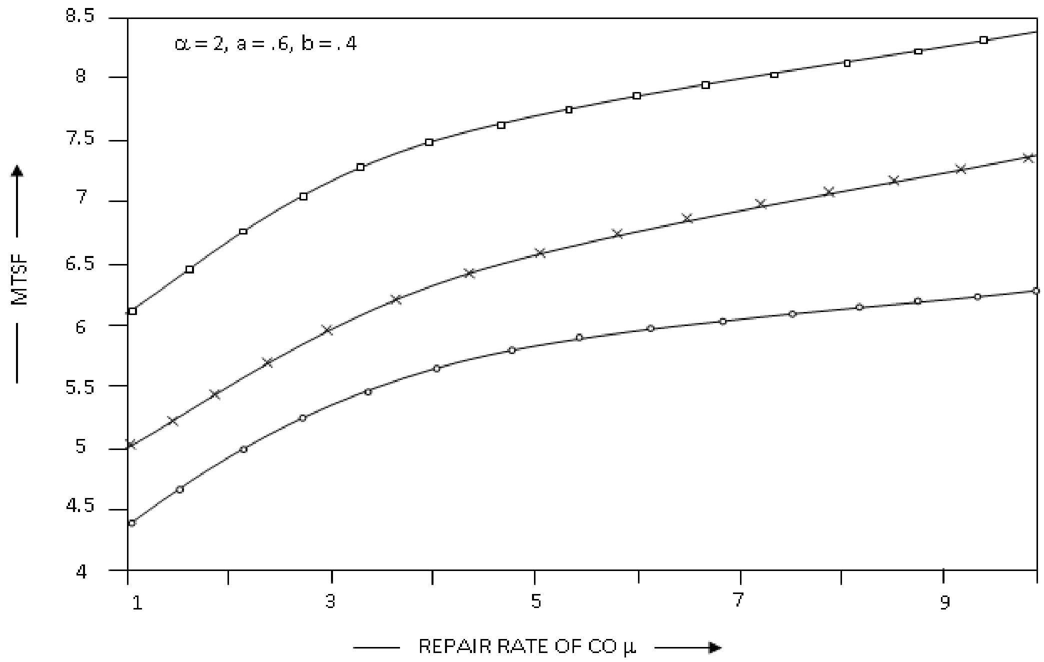

Fig. 3. (□)λ =.7, (×) λ =.8, (o) λ = .9

(iii) the expected total profit in a steady-state w.r.t. failure rate λ for different values of repair rate α (keeping other parameters as fixed) in Fig. 4.

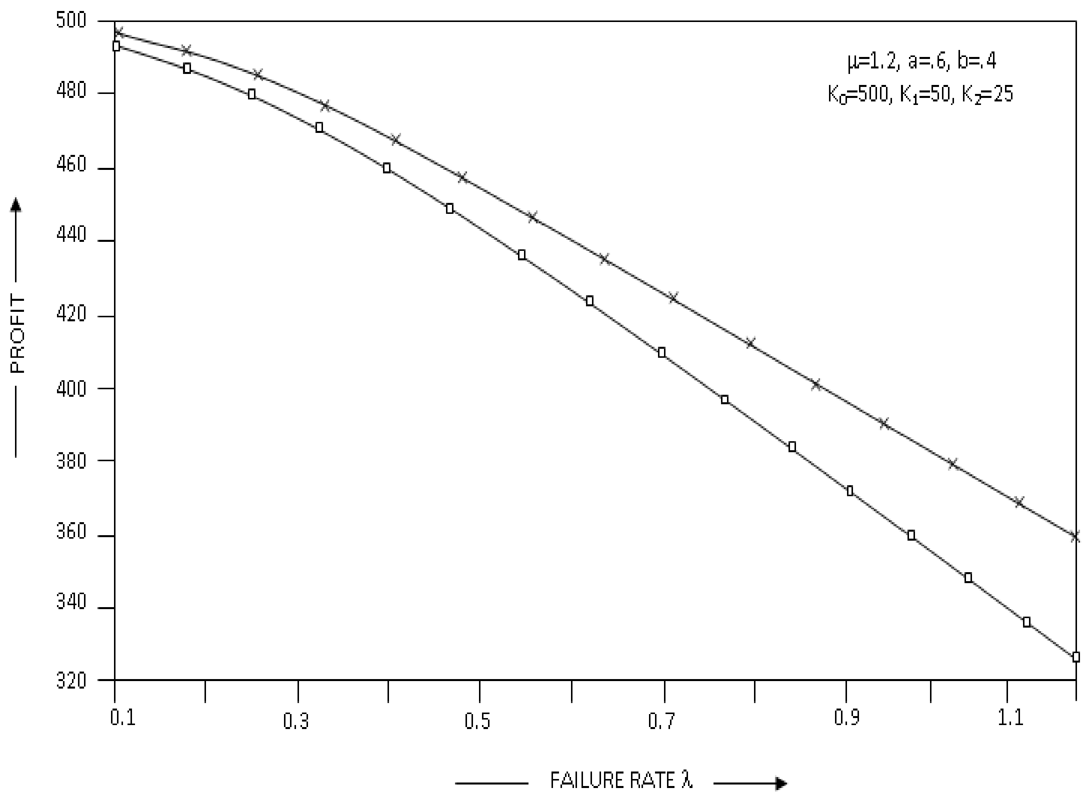

Fig. 4. (□)λ = 1.6, (×) α = .2

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